Unlocking Mathematics: New Technology for Deep and Early Understanding

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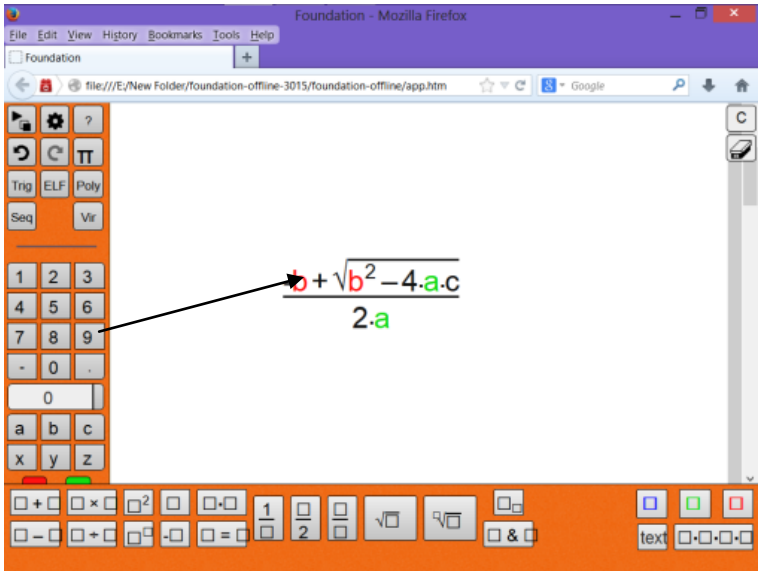
Remarkable results have been achieved with a tool that breaks new ground. It has application throughout the whole school Mathematics curriculum from Grade 1 addition and subtraction to Grade 12 Calculus.

Technology has enabled Mathematics to be presented as a process, something that research has shown to be much easier for children to understand (Sfard, 1989). This is achieved through movement on the screen. It could change the way that concepts are introduced in all grades.

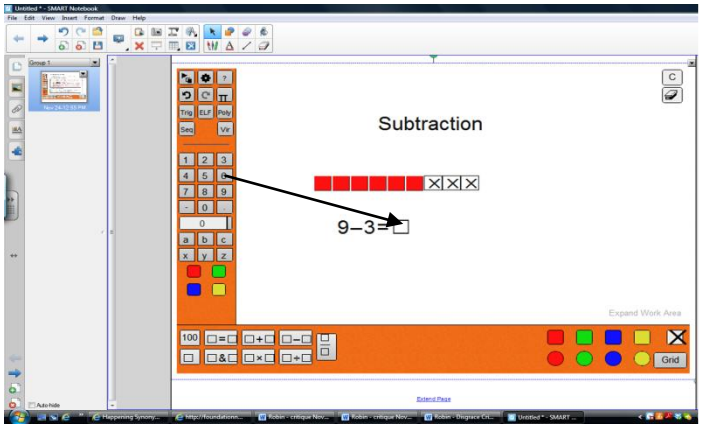
The original goal was to solve an interesting problem that has been around for 60 years or more and requires modern two dimensional screens for a solution. A simple way has been found for a computer to calculate directly from Mathematics notation, a facility that today's electronic boards can exploit well.



The tool is called “Foundation”, because it is a foundation upon which both arithmetic expressions and algebraic expressions can be built.

Numbers are on the left toolbar and can be dragged on to replace an algebraic variable (*as shown in the screen picture*), ready for calculation

Foundation expressions are built from functions such as the addition function (□+□) and the multiplication function (□×□). These are located on the bottom toolbar.



As the second screen picture shows, a number can also be dragged into a variable: 6

9 - 3 = ⬚

to give: 9 - 3 = 6

This example uses an alternative bottom toolbar with virtual manipulatives.

Although the tool had originally been intended for use in older grades, it has successfully been used in grades 1 and 2 to teach abstract Algebra concepts like expressions, equality and variables. It was also used to teach symmetry, basic arithmetic operations (*addition, subtraction, multiplication and division*), fractions, area, perimeter and data management.

In addition to calculating any school mathematical expression directly, functions appear to make Arithmetic a much better preparation for Algebra. Usage is expected to show that the introduction of functions in the elementary grades is an improved process for developing the literacy in Mathematics notation needed in high school.

The following were considered additional benefits of having used Foundation in the Grade 1 and 2 pilot classes: the ability to drag entire expressions helped students see them as whole entities rather than a sequence of individual parts; starting with the function ⬚+⬚ helped students see the plus sign as representing an operation (*the process of addition*) rather than as an arbitrary symbol, avoiding any confusion about which operational sign to use.

Effort has been devoted to keeping Foundation simple, consistent and mathematically intuitive, making it easy to learn. Apart from the need to contain nothing but mathematical notation, manipulatives and text, Foundation offers as much freedom as a blackboard or whiteboard. The teacher can place anything anywhere on the board and use it for any Mathematics lesson in any grade. The whole application of Foundation is under the control of the teacher or student, not the computer. It places no limitation on curriculum - nor does it influence

curriculum - it supports the whole curriculum and any lesson plans, both commercial and teacher-generated, in any grade.

The range of manipulatives and other features is being expanded grade by grade to maximize the potential benefits. High school functions are already in place, including polynomials, trigonometric functions, logarithmic functions, the exponential function and factorials.

Reference

Sfard, A. (1989). Transition from Operational to Structural Conception: the Notion of Function Revisited. G. Vergnaud, J. Rogalski, & M. Artigue (Eds), *Proceedings of the Thirteenth International Conference for the Psychology of Mathematics Education, 3,* 151-158. Paris; G.R. Didactique, CNRS.